1. Consider another version of the labor model we discussed from class. This time, the labor supplied by individuals will be given by

$$L_s(w) = w + b$$

and the labor demanded by firms by

$$L_d(w) = c - w.$$

$L_s(w)$ is hours supplied at wage w, $L_d(w)$ hours demanded at wage w, and $c > b > 0$.

a. What are the parameters to this model? What is the endogenous variable?

b. Choose values Draw a graph with hours on the x-axis and the wage on the y-axis. Graph labor supplied and labor demanded as a function of the wage w.

c. Solve for the equilibrium wage as a function of model parameters. Why is $c > b > 0$ important for the solution?

2. A firm rents capital K and hires labor L to maximize profit. The firm has production function

$$F(K, L) = AK^{\alpha_1} L^{\alpha_2}$$

with parameters α_1, α_2 both strictly between 0 and 1. The firm rents capital at interest rate R and pays labor wage W.

   1. Recall from class that if for any scalar $\lambda \geq 0$ we have

   $$F(\lambda K, \lambda L) = \lambda F(K, L),$$

   then the firm has constant returns to scale. Suppose $\alpha_1 + \alpha_2 = 1$, show that the firm has constant returns to scale.

   2. Write restrictions on the parameters α_1, α_2 such that the firm has increasing returns to scale. Write restrictions on the parameters α_1, α_2 such that the firm has decreasing returns to scale.

   3. Suppose now that $F(K, L) = AK^{\alpha} L^{1-\alpha}$. Write the profit maximization problem.

4. Explain why the profit that solves this maximization problem must be zero.

5. What is the marginal product of capital? What must the marginal product of capital be equal to?

6. What is the marginal product of labor? What must the marginal product of labor be equal to?

7. Suppose MPK $> R$. That is suppose the marginal product of capital is less than the interest rate R. First, using your answer for (5), show whether the marginal product of capital is increasing or decreasing in capital. Then, write whether or not the firm should rent more capital.

8. Suppose MPL $< W$. That is, suppose the marginal product of labor is less than the real wage W. First, using your answer for (6), show whether the marginal product of labor is increasing or decreasing in labor. Then, write whether the firm should increase or decrease its labor.

9. Suppose there's a new technology invented that enables the firm to produce more with the same resources. What parameter would you increase to represent this?

10. Consider the following scenario, let $W > 0$ and $R > 0$ be the real wage and interest rate respectively. Let $0 < A_1 < A_2$. Suppose firm one has TFP A_1 and produces with output function

$$F(K, L) = A_1 K^{\alpha} L^{1-\alpha}.$$

Suppose firm two has TFP A_2 and produces with output function

$$F(K, L) = A_2 K^{\alpha} L^{1-\alpha}.$$

Both firms hire until the marginal product of labor equals the wage W and the marginal product of capital equals the interest rate R. Using your solutions from the previous problems, choose and carefully explain which firm will hire more workers and rent more capital.

3. This question uses what we learned about the Solow model from class. Consider an economy in steady state with no population growth. Suppose an allied country experiences a catastrophe which leads to emigration, a fraction of which immigrate to the country in steady-state. This leads to a one-time population increase.
   Answer the following questions:

   (a) Draw a graph with capital (K) on the x-axis. On the y-axis, include lines for investment $\bar{s}Y$, depreciation (δK) and output (Y) for both before and after the one-time population change. Note that the variables you are asked to not are *not* per person values, but the levels for the entire economy.
   Hint: Depreciation δ will not change. You should have 5 lines on your graph.

   (b) Describe the *immediate* effect of this population increase on capital per capita and output per capita. As time passes and the country returns to the steady state, will these *per capita* values return to their old level, return to higher levels, or return to lower levels?

(c) Look in your textbook at graph 5.7(b). Note that the graph plots the evolution of output per person after a shock. Use this as a template to do the following:

i. Using your answers from part (a), graph the evolution of capital (K) and output (Y) after the one-time population increase. In your graph, $t = 0$ should be the time the population suddenly increases.

ii. Using your answers from part (b), graph the evolution of capital per person (k) and output per person (y) after the shock. In your graph, $t = 0$ should be the time the population suddenly increases.