1. Consider the Romer model we discussed in class (without capital) but with two modifications:

   1. Suppose that population growth is zero. For each time period t, suppose $N_t = \bar{N}$.
   2. Suppose that the creation of new ideas is proportional to the existing stock of ideas, such that
   $$\Delta A_{t+1} = A_t \bar{z} \ell \bar{N}.$$

   This means we have the following equations:

   $$\begin{aligned} L_y + L_a &= \bar{N} \\ Y_t &= A_t^{\gamma} L_y \\ \Delta A_{t+1} &= A_t \bar{z} L_a \\ L_a &= \ell \bar{N}. \end{aligned}$$

   (a) What is the growth rate in the stock of ideas? That is, find $g_{A_t} \equiv \frac{\Delta A_{t+1}}{A_t}$.

   (b) What is per-capita output at time t.

   (c) What is the growth rate of per-capita output?

For the following questions, suppose we have a one-time increase in the population from $\bar{N}$ to $\bar{N}'$.

(d) Does this have an immediate affect on output per-capita? Justify your answer with an explanation.

(e) How does this impact the growth rate of output per capita in the long-run?

2. We now will consider the same model except with capital. Our new equations are given by

$$\begin{aligned} L_y + L_a &= \bar{N} \\ Y_t &= A_t^{\gamma} K_t^{\alpha} L_y^{1-\alpha} \\ \Delta A_{t+1} &= A_t \bar{z} L_a \\ L_a &= \bar{\ell} \bar{N} \\ K_{t+1} &= \bar{s} Y_t + (1-\delta) K_t. \end{aligned}$$

where parameters $\alpha, \gamma, \ell, \bar{s}$, and δ are between 0 and 1 non-inclusive.

(a) Define a balanced growth path for this economy. Specifically, let $g_{A,t}$, $g_{K,t}$, and $g_{Y,t}$ be the growth rates for the above variables in time t. What does a balanced growth path imply for these variables?

(b) Using your above definition, and the capital accumulation equation, argue that it must be the case that $g_K = g_Y$. Conclude then, that this means the per capita growth rates are equivalent as well, so $g_k = g_y$. *Hint:* Look at the notes from class.

(c) Find the growth rate of output per person g_y in the balanced growth path as a function of only parameters. Compare this to what you obtained in 1.b.

3. Consider the human capital production structure we learned from class, given by

$$\begin{aligned} Y_t &= K_t^{\alpha}[(1-s_h)H_t]^{1-\alpha} \\ K_{t+1} &= s_K Y_t + (1-\delta)K_t \\ H_{t+1} &= (s_h H_t)^{\sigma} + (1-\delta)H_t \end{aligned}$$

(a). What are the endogenous variables in the model? What are the parameters? In class we interpreted each parameter. In your own words, write down what each parameter represents.

(b) Define a steady state. Find the steady state level of human and physical capital.

(c) Use these to find the steady state level of output.