1. Consider the extended Solow model which incorporates unmeasured capital from class:

$$\begin{aligned} Y_t &= K_t^{\alpha_K} Z_t^{\alpha_Z} (AL_t)^{1-\alpha_K-\alpha_Z} \\ K_{t+1} &= s_K Y_t + (1-\delta_K) K_t \\ Z_{t+1} &= s_Z Y_t + (1-\delta_Z) Z_t. \end{aligned}$$

   a. Define per-capita variables $y_t \equiv \frac{Y_t}{L_t}$, $k_t \equiv \frac{K_t}{L_t}$, and $z_t \equiv \frac{Z_t}{L_t}$. Transform the production function and both capital accumulation equations to be functions of these per-capita variables.

   b. Write down the equation for the steady state level of output per capita as a function of only model parameters and exogenous variables.

   c. Calculate the elasticity of steady state output per capita with respect to the physical capital investment rate (s_K). Do the same for the unmeasured capital investment rate (s_Z). Under what condition will a 1% increase in the unmeasured capital investment rate (s_Z) increase steady state output per capita more than a 1% increase in the physical capital investment rate (s_K)?

2. Consider the following household model:

$$\max_{c,n} \; c^{\alpha}(24-n)^{1-\alpha}$$
$$\text{such that } c = wn.$$

Note the similarity between our utility function here and the Cobb-Douglas production function. An agent that has this utility function is said to have Cobb-Douglas preferences. You can think of the term $24 - n$ as the number of hours in a day the agent is not working, which we call leisure.

a. Write the marginal rate of substitution. Set this equal to the real-wage (w).

b. Using this equation and the budget constraint, solve for labor supplied (n) as a function of model parameters and exogenous variables.

c. What is the elasticity of labor supplied (n) with respect to the real-wage (w)? Does the income effect dominate, the substitution effect dominate, or neither?

3. Consider the following model:

$$\max_{c,n} \frac{c^{1-\sigma}}{1-\sigma} - \chi \frac{n^{1+\frac{1}{\varepsilon}}}{1+\frac{1}{\varepsilon}}$$
$$\text{such that } (1+\tau_c)c = (1-\tau_n)wn.$$

Note that in addition to a tax on labor income, we also have a tax on consumption (an example of this is a sales tax). Remember the household takes the real wage (w), consumption tax (τ_c), and tax on labor (τ_n) as given.

a. Write the marginal rate of substitution. Instead of the real-wage, set this equal to $\frac{(1-\tau_n)w}{1+\tau_c}$. Using the intuition we discussed from class or a Lagrangian, explain why this equality holds at the optimum pair (c, n).

b. Using this equation and the budget constraint, solve for labor supplied (n) as a function of model parameters and exogenous variables.

c. Pick a city in a location that has both a sales and income tax (in any country of your choosing). Look online for a typical sales tax rate and for a typical marginal income tax for middle-income earners. Using your findings, write the name of the city and fill in the following table:

Parameter	Value
σ	2
ε	0.5
χ	1
w	30
τ_c	
τ_n	

Now, divide labor supplied without an income tax by labor supplied with an income tax (holding your consumption tax constant at the rate you entered in the table above). Holding all else constant, use your expression for labor supplied to write down which tax has the biggest impact.