Equilibrium

Graham Lewis

University of Minnesota

June 2025

Last lecture

- We modeled labor supply with a representative household that obtained utility from consumption and disutility from working.
- We added a tax on labor income and a transfer from the government.
- We added a government budget constraint, which said that the transfers had to equal the tax revenue from labor income.

Motivation

- We now have a model for how labor is supplied in the economy (representative household).
- We also have a model for how labor is demanded in the economy (representative firm).
- Today we will discuss an equilibrium, where labor supply equals labor demanded.

Household Side

- The household, taking the wage as given, maximized utility subject to the budget constraint

$$\max_{c,n} \; u(c) - v(n)$$
$$\text{such that } c = wn.$$

- This meant that given a real-wage (w), the household choose labor supply (n) and consumption (c) that both maximized its utility and was affordable.
- For each wage w, could calculate the labor supplied (n).

Firm Side

- Consider a representative firm that, taking the real-wage (w) as given, chooses labor demanded L to maximize its profit:

$$\max_{L} AL^{\theta} - wL.$$

- We discussed in the production lecture that the firm will hire labor until the additional gains in revenue it has from hiring labor equals the cost, given by

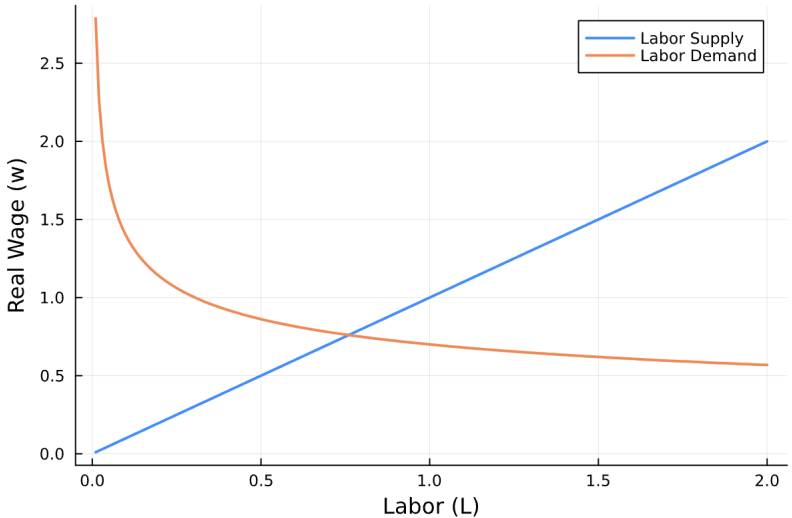
$$\begin{aligned} \text{MPL} &= w \\ \theta A L^{\theta-1} &= w. \end{aligned}$$

- So, for each wage w, we will have labor demanded by the firm.

Equilibrium

- For each wage value (w) we have labor supplied and labor demanded.
- We will search for the wage (w) that clears the market.
- The labor market "clears" when the amount of labor supplied equals the amount of labor demanded.
- This is driven by the price of labor (w). If the real-wage (w) is too high, then labor supplied is greater than labor demanded. If the real-wage is too low, then labor demanded (w) is greater than labor supplied.

Equilibrium



Equilibrium Example

- We will solve for the equilibrium wage for a problem will the following setup:
- Suppose a representative household, taking the real-wage (w) as given, solves

$$\max_{c,n} \; c - \frac{n^2}{2}$$
$$c = wn + \Pi.$$

- Suppose a representative firm, taking the real-wage (w) as given, solves

$$\max_{L} AL^{\theta} - wL$$

where $0 < \theta < 1$.

Household Side

- The household faced

$$\max_{c,n} \; c - \frac{n^2}{2}$$
$$c = wn + \Pi.$$

- We have a new term Π we have not seen before, in this model we will assume firm profits (which so far in the course have been zero) will be equally distributed across households.
- Note that our firm problem did not have a CRS production function, so we need to account for the profits that come from the firm production problem.
- The household takes these as given, we will focus on the labor supply decision.

Household Side

- We had

$$\max_{c,n} \; c - \frac{n^2}{2}$$
$$c = wn + \Pi.$$

- The MRS condition will be

$$\begin{aligned} \text{MRS} &= w \\ -\frac{U_n}{U_c} &= w \\ n &= w. \end{aligned}$$

- So, the labor supplied at wage w, which we'll denote $L^s(w)$, will be w.

Firm Side

- Our firm problem was given by

$$\max_L AL^\theta - wL.$$

- From our production lecture we know the firm will hire until the marginal product of labor equals the real-wage w:

$$\theta A L^{\theta-1} = w$$

$$L = \left(\frac{\theta A}{w}\right)^{\frac{1}{1-\theta}}.$$

- We now will have an equation for labor demanded, which we'll denote $L^d(w)$.

Equilibrium

- In an equilibrium, labor supplied will equal labor demanded.
- We can set

$$\begin{aligned} L^d(w) &= L^s(w) \\ \left(\frac{\theta A}{w}\right)^{\frac{1}{1-\theta}} &= w \\ \frac{\theta A}{w} &= w^{1-\theta} \\ (\theta A)^{\frac{1}{2-\theta}} &= w. \end{aligned}$$

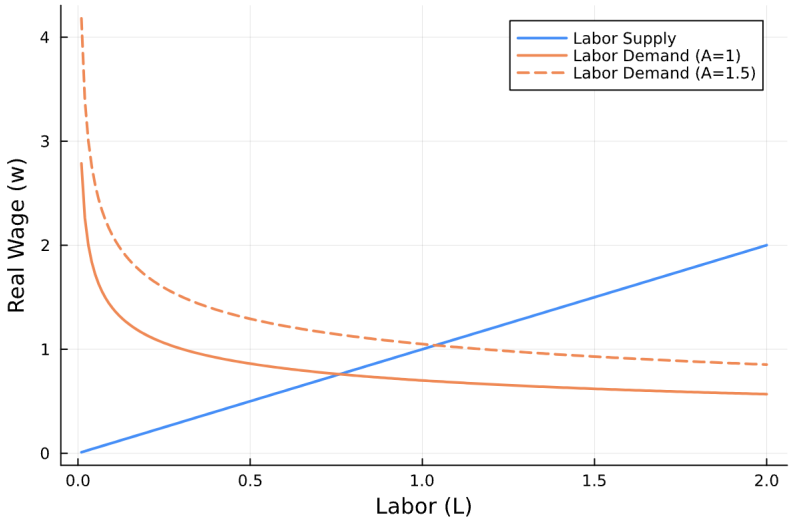
- We find that the equilibrium wage is given by $w = (\theta A)^{\frac{1}{2-\theta}}$.

Equilibrium

- We had $w = (\theta A)^{\frac{1}{2-\theta}}$.
- From this equation, we can see that an increase in TFP (A) would increase the equilibrium wage.
- This increase in the equilibrium wage would come from a shift in the labor demanded curve to the right, given our labor demand

$$L^d(w) = \left(\frac{\theta A}{w}\right)^{\frac{1}{1-\theta}}.$$

Equilibrium[1]



[1] $\theta = 0.7$

Household Side

- Suppose we add something that shifts labor supplied.
- Consider the same model we had now with a labor income tax and ocean disposal

$$\max_{c,n} c - \frac{1}{2}n^2$$
$$\text{such that } c = (1-\tau_n)wn + \Pi.$$

- The MRS is given by

$$\begin{aligned} \text{MRS} &= (1-\tau_n)w \\ -\frac{U_n}{U_c} &= (1-\tau_n)w \\ n &= (1-\tau_n)w. \end{aligned}$$

- So labor supplied $L^s(w) = (1-\tau_n)w$.

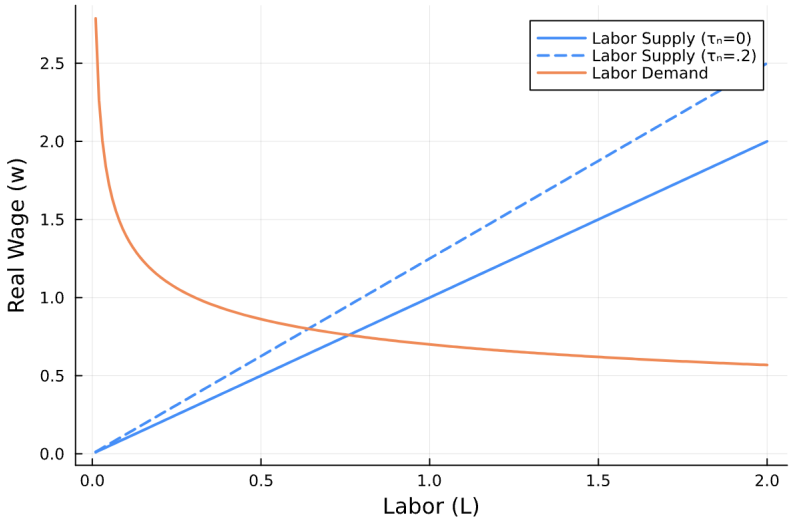
Household Side

- We'll keep the firm problem the same, so to solve for the equilibrium wage we'll set

$$\begin{aligned} L^d(w) &= L^s(w) \\ \left(\frac{\theta A}{w}\right)^{\frac{1}{1-\theta}} &= (1-\tau_n)w \\ \theta A &= (1-\tau_n)^{1-\theta} w^{2-\theta} \\ \left(\frac{\theta A}{(1-\tau_n)^{1-\theta}}\right)^{\frac{1}{2-\theta}} &= w. \end{aligned}$$

- Note that we had $L^s(w) = (1-\tau_n)w$, so an increase in the tax rate on labor income reduces labor supplied, which drives up the wage in equilibrium.

Household Side[2]



[2] $A = 1.0$

Discussion & Extensions

- We have a model of labor supply, in which a representative household chooses consumption (c) and labor (n) to maximize utility subject to a budget constraint.
- We have a model of labor demand, in which a representative firm hires labor until MPL $= w$.
- We discussed an equilibrium in which prices adjusted to ensure that supply equaled demand.
- The basic structure of the model we studied here can be used in a vast number of different environments.

Multiple Time Periods

- We can consider the model with multiple time periods.
- Next mini you will see a model in two time periods, in which the household must make tradeoff decisions between consuming today and consuming tomorrow.
- In models with multiple periods, labor supplied and labor demanded, the wage (w) still adjusts each period to clear the market.

Physical Capital

- Next mini you will see how capital is supplied in the economy.
- In models with capital, the price of capital (r) will adjust until the capital supplied equals capital demanded.
- The micro-foundation of our model, in which the household has a budget constraint and utility function, will also not change.

Extensions

- The basic framework we used here has been limited in its complexity, the foundation and ideas behind them are generalizable and can be used to study a wide variety of problem, including
  1. Human capital
  2. Entrepreneurship
  3. Physical health
  4. Expansions in government policy that include all of the above.

Summary

- We've seen how labor markets clear via wage adjustments.
- We've modeled how changes in taxes (τ_n) TFP (A) affect labor supplied and labor demanded in equilibrium.
- We discussed all the extensions where we can apply this framework.