Ideas

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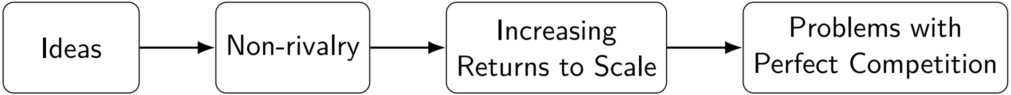
Motivation

- Last class we saw that while the Solow model provided valuable insights, it was not a complete picture.
  1. There was still unexplained differences in growth between countries.
  2. The Solow model did not yield long-run growth.
- In 1990, Paul Romer (2018 Nobel Laureate) proposed a distinction between
  - **Objects:** Land, machines, commodities, etc.
  - **Ideas:** Institutions, patents, management techniques, etc.

Economics of Ideas



Ideas

- You can think of ideas as the ways to arrange raw materials in ways that are economically useful.
- With new ideas, we can organize limited resources into more efficient ways, producing more with the same amount.

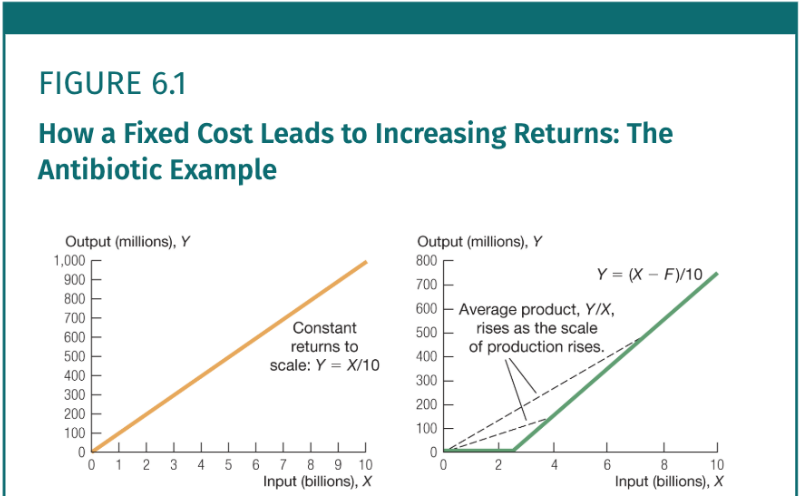
Non-rivalry

- **Rival**: One person's use of the good affects the availability of that good.
  Example: Parking spots, cellphones
- **Non-rival**: Infinitely usable goods.
  Song lyrics, jet airplane assembly instructions, broadcast television
- Important note: A restriction on the use of an idea (patent protection) does not affect its status as a non-rival good.

Increasing Returns

- Example: Antibiotic production
  1. For an already developed drug, we can model production with a CRS production function.
     - If a factory with workers can produce 100 doses a day, then building an identical factory next door should approximately double output. Doubling input doubles output.
  2. To develop a new drug, there is an R&D cost that must be paid before production.
     - If a company pays \$2.5 billion to develop a drug and can produce each dose at \$10, then doubling inputs from \$2.5 billion to \$5 billion would more than double production.

Example: Antibiotic Production



(a) Constant returns to scale: $Y = X/10$

(b) Increasing returns from fixed cost: $\bar{F} = 2.5$ billion

Incorporating Ideas

- In our previous lectures we had

$$F(K, L) = AK^{\alpha}L^{1-\alpha}.$$

- We saw that if you doubled the inputs, then you double the output

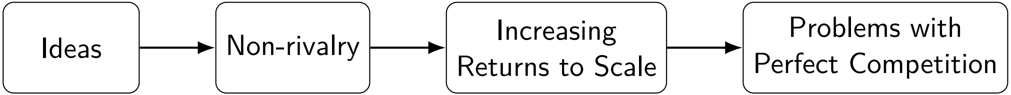
$$F(2K, 2L) = A(2K)^{\alpha}(2L)^{1-\alpha} = 2(AK^{\alpha}L^{1-\alpha}).$$

Incorporating Ideas

- Consider
$$F(K, L, A) = AK^{\alpha}L^{1-\alpha}$$
That is, TFP A is now an input.
- If you double inputs, you more than double the output
$$F(2K, 2L, 2A) = 2A(2K)^{\alpha}(2L)^{1-\alpha} = 4(AK^{\alpha}L^{1-\alpha}).$$
- Similar to our antibiotics example, this new production function exhibits constant returns to scale in K and L but increasing returns to scale in ideas and objects taken together.

Economics of Ideas

- Recall our diagram



- Our last point was a problem with perfect competition.

Problems With Pure Competition

- Suppose, as in a world with perfect competition and firms must sell at price equals marginal costs.
- A firm that has already made the drug can sell it at $10 a dose.
- A firm thinking of investing in a new drug will not invest, as it will not be able to recoup the fixed cost of the investment.
- The point is more general, any new innovation that requires a fixed cost will not be pursued in an environment where firms must sell at marginal cost.

Profit Motive

- When new ideas are invented, there is a fixed cost to produce this new way to organize resources.
- After this, production occurs with constant returns to scale.
- In order for the pursuit of this advancement to have been worthwhile, there must be a difference between price and marginal cost.
- This means if we want innovation, our market cannot have pure competition.

Encouraging Innovation

- Patent and copyright systems offer a way to protect innovations through offering innovators monopoly power for 20 years in exchange for the innovator making the design behind their idea public.
- Trade secrets offer the opportunity to use proprietary methods while not sharing the design, though they do not offer monopoly in the market.
- Government also provides incentives for research through organizations like the National Science Foundation and the National Institutes of Health.
- Prizes have also been a solution. Charles Lindbergh's nonstop flight from NYC to Paris (the first transatlantic flight) is credited with spurring technological process in aviation. The flight was done in part to obtain a $25,000 prize.

Case Study

- An open area of debate is the extent to which intellectual property of industrialized countries should be observed in developing countries.
- Supporters of strict adherence claim that ignoring these rights discourages multinational firms locating to these countries and prevents the transfer of new technologies.
- Supporters of looser adherence claim that poorer countries can obtain pharmaceuticals and other essential technologies at an affordable price if they are not forced to pay the premiums protected products charge.

Model Requirements

- Based on the previous slides, to incorporate ideas into a model, we will need
  1. A distinction between ideas and objects
  2. Increasing returns to ideas
- The Romer model meets both requirements.

Romer Model

- Let N_t be the total population of workers in time period t.
- We will split these workers into two categories: those who work to produce ideas and those to work to produce goods.
- L_{yt} : Those who work to produce goods.
- L_{at}: Those who work to produce ideas.

$$L_{yt} + L_{at} = N_t.$$

Romer Model

- Goods will be produced with the production function

$$Y_t = A_t^{\gamma} L_{yt}.$$

- Notice that this function exhibits constant returns to objects (labor) and increasing return to objects and ideas.
- For simplicity, we are not yet adding capital to the model.

Romer Model

- We have modeled the workers who produce goods (L_{yt}). We now need to model how ideas are produced.
- Let's assume each worker in the idea sector (L_{at}) produces $\bar{z}$ new ideas each year.
- Each year there are $\bar{z}L_{at}$ new ideas and no depreciation, so

$$\Delta A_{t+1} = \bar{z}L_{at}.$$

Romer Model

- As we did when adding population growth to the Solow model, we will assume the population grows at a constant rate.
- That is, the population will grow at some $\bar{n} > 0$ where

$$\frac{\Delta N_{t+1}}{N_t} = \bar{n}.$$

Romer Model

- So far we have five unknowns L_{yt}, L_{at}, N_t, and A_t.
- We have four equations

$$
\begin{aligned}
L_{yt} + L_{at} &= N_t \\
Y_t &= A_t^{\gamma} L_{yt} \\
\Delta A_{t+1} &= \bar{z} L_{at} \\
\frac{\Delta N_{t+1}}{N_t} &= \bar{n}.
\end{aligned}
$$

- In order to fully solve the model, we need the same number of unique equations as unknowns.
- We will add a simplifying equation

$$L_{at} = \bar{\ell} N_t$$

which says the share of workers who work in the idea sector is constant.

Output per person

- Recall from last lecture even with constant growth of the labor supply, we found that output per person did not grow over the long run in the Solow growth model.
- Output per person in the Romer model is given by

$$
\begin{aligned}
y_t &\equiv \frac{Y_t}{N_t} \\
&= A_t^{\gamma} \frac{(1-\bar{\ell})N_t}{N_t} \\
&= A_t^{\gamma}(1-\bar{\ell}).
\end{aligned}
$$

- Output per person depends on the **total** stock of knowledge.
- In the Solow model we found that output person depended on on the capital **per person**.

Output per person

- We found that

$$y_t = A_t^{\gamma}(1 - \bar{\ell}).$$

- The reason that an increase in the total stock of knowledge increases the output per person is that ideas are non-rivalrous.
- If you add an idea, it can be used by everyone.
- This idea enables the model to generate long-run growth in output per person.

Balanced Growth Path

- As mentioned earlier, our economy is growing in the long run.
- The solution to our model is a balanced growth path.
- A balanced growth path is when variables grow at a constant rate over time.
- An implication of a balanced growth path is the ratio between variables (such as capital-output) remain constant.
- It is important to note that these variables need not grow at the *same* rate, but they need to grow at a constant rate.

Output Per Person

- From your previous recitation, we know that the growth rate of a variable raised to a power is that power times the variable's growth rate. Thus, since

$$y_t = A_t^{\gamma}(1-\bar{\ell})$$

$$g_y^* = \gamma g_A^*.$$

- Since the stock of knowledge is proportional to the number of researchers, then the growth rate of knowledge equals the growth rate of researchers.

- This means that $g_A^* = g_{L_a}^* = \bar{n}$ since $L_{a_t} = (1-\bar{\ell})N_t$ and

$$\frac{\Delta N_{t+1}}{N_t} = \bar{n}.$$

- Putting these together means

$$g_y^* = \gamma \bar{n}$$

Output Per Person

- We had
$$g_y^* = \gamma \bar{n}.$$
- We have discussed at length about the source of growth in the economy.
- According to this equation, the parameter that governs the degree of increasing returns to scale (γ) and the growth rate of the number of researchers $(\bar{n}$ since this grows the same as the population$)$ determine the growth rate of output per person.
- More people means more ideas, each idea benefits everyone.

Global framework?

- Luxembourg has way less researchers than the United States but has a higher output per person. This is because the benefits to new ideas applies globally, not just to each individual country.
- We can think of the Romer model as a model for the world, not necessarily a model for each country.

Shift in productivity

- Suppose workers get more productive.
- It can be shown that output per person is given by

$$y_t = (1-\bar{\ell})\bar{\ell}^{\gamma}\left(\frac{\bar{z}N_t}{\bar{n}}\right)^{\gamma}$$

- We also had

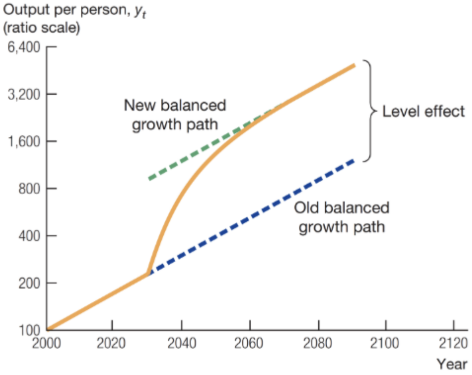
$$g_y = \gamma\bar{n}.$$

- Thus, we see that an increase in the productivity of workers $(\bar{z})$ leads to a level effect on output per person but not an effect in the growth rate of output.

Output per person

FIGURE 6.3

Experiment #1: A Permanent Rise in Research Productivity, $\bar{z}$



Transition Dynamics

- We see that the new balanced growth path grows at the same rate as the old one, but is shifted upwards from the level effect.
- Note that the same transition dynamics as those from the Solow model occur.
- The further the economy is from the steady state level, the faster the economy moves towards that level.

Shift in research share

- Suppose the share of researchers increases.
- We had
$$y_t = (1-\bar{\ell})\bar{\ell}^{\gamma}\left(\frac{1}{\bar{n}}\right)^{\gamma}(\bar{z}N_t)^{\gamma}.$$
- Notice that increasing $\bar{\ell}$ will increase $\bar{\ell}^{\gamma}$ and decrease $(1-\bar{\ell})$.
- Increasing the number of researchers increases the number of ideas generated but decreases the number of people producing the consumptive good.
- The relation between y_t and ℓ is hump-shaped. Too many people in research and you have zero production ($\bar{\ell} \to 1$) and too few researchers and the number of ideas is low ($\bar{\ell} \to 0$).

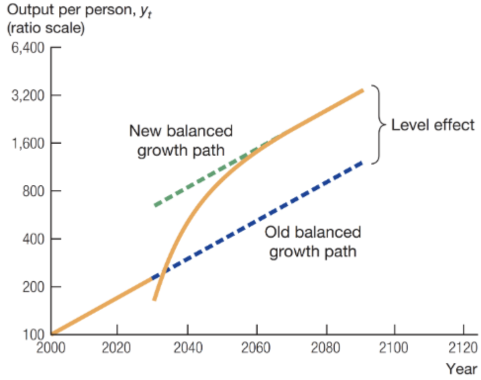
Rise in Research Share

- Suppose an increase in $\bar{\ell}$ raises the output per person.
- We know that the end destination of our output per person will be higher.
- However, the path is not as straightforward as previous examples.
  - There are initially less people to produce goods.
  - The people moving to research stop producing before generating any ideas.

Rise in Research Share

FIGURE 6.4

Experiment #2: A Permanent Rise in the Research Share, $\bar{\ell}$



Rise in Research Share

- Notice the rise in the research share doesn't initially drive a rise in the stock of ideas.
- However, a rise in the research share does initially lower production.
- These two together lead to the initial drop, which rises after the increase in researchers lead to growth of the stock of ideas.
- Putting more of the labor force into research temporarily reduces production.

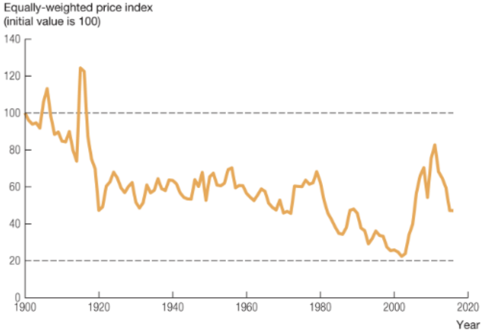
Case Study: Environmental resources

- Consider the following: When a natural resource like coal was first used, supply was untouched and demand small.
- Over time, demand grew and the global supply decreased.
- What would you expect to happen to the price of coal over time?

Case Study: Environmental Resources

FIGURE 6.5

The Real Price of Industrial Commodities



Source: Based on data from Global Financial Data and FRED for aluminum, copper, zinc, lead, iron ore, and coal.

Case Study: Environmental Resources

- This surprising trend seems counterintuitive, how can the price fall when existing supplies have decreased and demand increase?
- While the story about demand was correct, our story of supply was more nuanced.
- New technologies allowed us to both discover previously unobserved supplies of these resources and extract pockets thought to be inaccessible.
- Evidently, these technological forces helped lead to a supply force that outweighed the massive increase in demand.

Adding Capital

- How does the growth rate of output per person change when we add capital?
- We will assume a production function of

$$Y_t = A_t^{\gamma} K_t^{\alpha} L_{yt}^{1-\alpha}.$$

- Note that this is similar to the production function we studied but with a returns to scale γ parameter.
- We will assume capital accumulates as in the Solow model

$$K_{t+1} = \bar{s} Y_t + (1-\delta) K_t.$$

Romer with Capital

- We now have 6 unknowns $(Y_t, A_t, K_t, L_{yt}, L_{at}, N_t)$ and 6 equations

$$
\begin{aligned}
Y_t &= A_t^{\gamma} K_t^{\alpha} L_{yt}^{1-\alpha} \\
K_{t+1} &= \bar{s} Y_t + (1-\delta) K_t \\
\Delta A_{t+1} &= \bar{z} L_{at} \\
N_t &= L_{yt} + L_{at} \\
L_{at} &= \bar{\ell} N_t \\
\bar{n} &= \frac{\Delta N_{t+1}}{N_t}.
\end{aligned}
$$

- This is very similar to the Solow model, however what was TFP (A_t) is now the stock of ideas and no longer constant.

Balanced Growth Path

- We will look for a balanced growth path. That is, a path where output, capital, and the stock of ideas grow at constant rates.
- We want to compare the growth rate of output per person in this environment to the one we had without capital.
- First, let's define output per person

$$
\begin{aligned}
y_t &\equiv \frac{Y_t}{N_t} \\
&= A_t^{\gamma} K_t^{\alpha} ((1-\bar{\ell})^{1-\alpha} (N_t)^{-\alpha} \\
&= A_t^{\gamma} k_t^{\alpha} (1-\bar{\ell})^{1-\alpha}
\end{aligned}
$$

where $k_t \equiv \frac{K_t}{N_t}$ is capital per person.

Growth Rate

- From chapter 3, we know
  1. The growth rate of several variables is the sum of each of the variable's growth rates.
  2. The growth rate of a variable raised to a power is that power times the variable's growth rate.
- Then we know since

$$y_t = A_t^{\gamma} k_t^{\alpha} (1 - \bar{\ell})^{1-\alpha}$$

  we get

$$g_{y_t} = \gamma g_{A_t} + \alpha g_{k_t}.$$

- Luckily, all the work we did above applies here and we have

$$g_{A_t} = \bar{n}.$$

- To figure out the growth rate of output per person g_{y_t} we need to solve for the growth rate of capital per person g_{k_t}.

Growth Rate of Capital

- We can figure out the growth rate of capital by

$$
\begin{aligned}
g_{K_t} &= \frac{\Delta K_{t+1}}{K_t} \\
&= \frac{\bar{s}Y_t + (1-\delta)K_t - K_t}{K_t} \\
&= \bar{s}\frac{Y_t}{K_t} - \delta.
\end{aligned}
$$

- Recall in a balance growth path the growth rate of capital g_{K_t} must be constant over time. Also, note that $\bar{s}$ and δ never change.
- This means that $\frac{Y_t}{K_t}$ never changes as well, so $g_{Y_t} = g_{K_t}$.

Growth Rate of Output per Person

- From the previous slide, we know the growth rate of output and capital are the same ($g_{Y_t} = g_{K_t}$).
- This means that the growth rate of output per person and capital per person are the same, so

$$g_{y_t} = g_{k_t}.$$

- So we can transform our equation for the growth rate of output per person as

$$\begin{aligned} g_{y_t} &= \gamma\bar{n} + \alpha g_{k_t} \\ &= \gamma\bar{n} + \alpha g_{y_t} \\ g_{y_t} &= \frac{1}{1-\alpha}\gamma\bar{n} \end{aligned}$$

Comparison

- In the Romer model without capital, we had

$$g_{y_t} = \gamma \bar{n}.$$

- In the Romer model with capital, we have

$$g_{y_t} = \frac{1}{1-\alpha} \gamma \bar{n}.$$

- Since $0 < \alpha < 1$, $\frac{1}{1-\alpha} > 1$, which means output per person grows faster in the Romer model with capital than the Romer model without capital.

- Why is this the case?

Channels from Capital

- Recall that in the Solow model an increase in TFP raised the steady state level of capital which raised output per person.
- This same logic holds here, the increase in the stock of ideas that comes from research increases output per person directly (direct effect) *and also* increases the level of capital, which increases the output per person (indirect effect).
- Capital cannot serve as an engine of economic growth itself, but it can *amplify* the economic growth.

Growth Accounting

- We discussed how the growth of output per person could come from changes in the stock of ideas, increases in capital per person, and changes in the labor composition.
- Using the growth accounting rules from Chapter 3, we see that we can turn

$$\frac{Y_t}{L_t} = \frac{A_t K_t^{\alpha} L_{y_t}^{1-\alpha}}{L_t}$$

into

$$g_{Y_t} - g_{L_t} = \alpha(g_{K_t} - g_{L_t}) + (1-\alpha)(g_{L_{y_t}} - g_{L_t}) + g_{A_t}.$$

- The growth rate of output per hour can be decomposed into the growth rate of capital per hour, changes in the labor composition, and the growth rate of TFP.

Growth Accounting

TABLE 6.2

Growth Accounting for the United States

	1948–2021	1948–1973	1973–1995	1995–2003	2003–2021
Output per hour, Y/L	2.4	3.3	1.5	3.2	1.8
Contribution of K/L	0.9	1.0	0.8	1.4	0.8
Contribution of labor composition	0.2	0.2	0.2	0.3	0.3
Contribution of TFP, A	1.2	2.1	0.5	1.5	0.7

The table shows the average annual growth rate (in percent) for different variables.

Source: Bureau of Labor Statistics, *Multifactor Productivity Trends.*

Growth Accounting

- Note the slowdown in output per hour between 1948-1973 and 1973-1995.
- Using our decomposition from above, we see that this slowdown was mainly driven by the contribution from TFP (A), which dropped precipitously.
- Economists have posed many answers to why this occurred, including the oil shock of the 1970's, a decline in spending on research and development, and the rise of services over manufacturing as the dominant sector in the economy.

Moving Forward

- The Romer model helped us understand why countries grow in the long-run.
- New ideas and inventions can drive long-term growth, while capital amplifies the growth.
- While we were able to use the Romer model to understand proponents of growth better, we were not able to use it to answer why some countries grow faster than others. Over the next few lectures, we will add complexity to our consideration of capital.