Unmeasured Capital

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Review

- In previous lectures we established the inability of our production model to account for cross-country differences in output per person.
- We used this as a motivation to expand beyond our discussion of physical capital.
- We saw that while human capital helped us better account for these differences, it was not sufficient.
- In today's lecture we will expand our definition even further, considering as investment any payments now that give returns in the future.[1]

[1] See chapters posted from "Barriers to Riches" by Edward Prescott and Stephen Parente on Canvas.

Unmeasured capital

- With this expanded notion of capital, investment rates will play a large role in determining steady state output per person.
- Investment rate differences will not only drive differences in physical capital accumulation, but unmeasured capital accumulation as well.
- Thus, differences in investment rates can compound from physical and unmeasured capital.

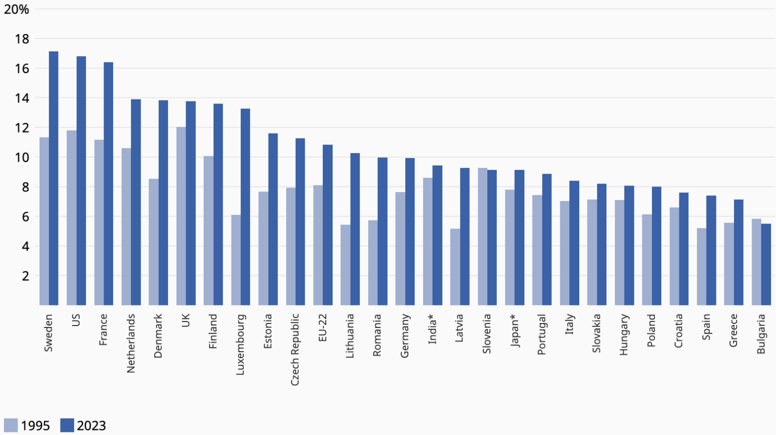
Unmeasured Capital: Examples

1. R&D expenditures
2. Maintenance and repairs that extend asset life
3. Software development
4. Human capital investment on the job

Unmeasured Capital: Measurement Error

- Countries often lack comprehensive data on intangible investments such as informal training, entrepreneurial effort, and R&D development.
- Official investment figures can understate true investment, especially in low-income countries.
- This makes cross-country comparisons tricky, as some high income countries measure intangibles considerably better than low-income countries.

Intangible Investment Differences[2]



Notes: *For India, shares are for the years 2011 and 2020, respectively, owing to the unavailability of data before 2011 and beyond 2020. Data for India exclude the informal sector. For Japan, shares are for the years 1995 and 2021, respectively, owing to the unavailability of data beyond 2021. See note of Figure 1 for definition of EU-22.

[2]Source: WIPO-LBS Global INTAN-Invest Database, June 2024

Intangible Investment

- As economies shift toward more innovation-driven growth, a larger share of output is allocated to research and development (R&D) rather than traditional physical investment.
- R&D spending creates patents, software, and product designs, all of which are forms of intangible capital.
- Incorporating alternative forms of capital helps us better model differences in productivity and income across nations.

Unmeasured capital

- Consider an extended Solow model with physical capital (K), unmeasured capital (Z), and labor (L).
- The production function is

$$Y_t = K_t^{\alpha_K} Z_t^{\alpha_Z} (AL)^{1-\alpha_K-\alpha_Z}.$$

- Note that this formulation reflects the idea that A augments labor directly. Doubling TFP is similar to doubling the workforce.

Capital accumulation equation

- Our physical capital and unmeasured capital accumulation equations are given by

$$K_{t+1} = (1 - \delta_K)K_t + s_K Y_t$$
$$Z_{t+1} = (1 - \delta_Z)Z_t + s_Z Z_t.$$

- The accumulation of physical and unmeasured capital is governed by the depreciation rate δ and the investment rate s.
- Note that compared to our consideration of human capital, we allow for different depreciation rates between unmeasured capital and physical capital.

Per Worker Transformation

- As in previous lectures, we'll suppose a constant population $\bar{L}$, defining per worker variables $y_t \equiv \frac{Y_t}{\bar{L}}$, $k_t \equiv \frac{K_t}{\bar{L}}$, and $z_t \equiv \frac{Z_t}{\bar{L}}$.
- We can rewrite our production function as

$$\begin{aligned}
Y_t &= K_t^{\alpha_K} Z_t^{\alpha_Z} (AL)^{1-\alpha_K-\alpha_Z} \\
\frac{Y_t}{\bar{L}} &= \frac{K_t^{\alpha_K} Z_t^{\alpha_Z} (AL)^{1-\alpha_K-\alpha_Z}}{\bar{L}} \\
y_t &= \left(\frac{K_t}{\bar{L}}\right)^{\alpha_K} \left(\frac{Z_t}{\bar{L}}\right)^{\alpha_Z} \left(A\frac{\bar{L}}{\bar{L}}\right)^{1-\alpha_K-\alpha_Z} \\
y_t &= k_t^{\alpha_K} z_t^{\alpha_Z} A^{1-\alpha_K-\alpha_Z}.
\end{aligned}$$

Per Worker Transformation

- We'll do the same for the capital accumulation equations, given by

$$
\begin{aligned}
K_{t+1} &= (1-\delta_K)K_t + s_K Y_t \\
\frac{K_{t+1}}{\bar{L}} &= \frac{(1-\delta_K)K_t + s_K Y_t}{\bar{L}} \\
k_{t+1} &= (1-\delta_K)k_t + s_K y_t.
\end{aligned}
$$

and the second is given by

$$
\begin{aligned}
Z_{t+1} &= (1-\delta_Z)Z_t + s_Z Y_t \\
\frac{Z_{t+1}}{\bar{L}} &= \frac{(1-\delta_Z)Z_t + s_Z Y_t}{\bar{L}} \\
z_{t+1} &= (1-\delta_Z)z_t + s_Z y_t.
\end{aligned}
$$

Steady State

- We can carry over our definition of a steady state from the Solow model. Here, a steady state is given by a triplet (y_{ss}, k_{ss}, z_{ss}) that solves

$$
\begin{aligned}
y_{ss} &= k_{ss}^{\alpha_K} z_{ss}^{\alpha_Z} A^{1-\alpha_K-\alpha_Z} \\
k_{ss} &= (1-\delta_K)k_{ss} + s_K y_{ss} \\
z_{ss} &= (1-\delta_Z)z_{ss} + s_Z y_{ss}.
\end{aligned}
$$

- Like our Solow model, in the steady state we know physical capital depreciation must equal physical capital investment and unmeasured capital depreciation must equal unmeasured capital investment.

- Note that we have three equations and three unknowns.

Steady state

- To solve for steady state use the capital accumulation equations to get

$$k_{ss} = \left(\frac{s_K}{\delta_K}\right) y_{ss}$$
$$z_{ss} = \left(\frac{s_Z}{\delta_Z}\right) y_{ss}.$$

- Plugging these into our equation for steady state output per capita gives us

$$y_{ss} = A^{1-\alpha_K-\alpha_Z} k_t^{\alpha_K} z_t^{\alpha_Z}$$
$$y_{ss} = A^{1-\alpha_Z-\alpha_K} \left(\frac{s_K}{\delta_K}\right)^{\alpha_K} \left(\frac{s_Z}{\delta_Z}\right)^{\alpha_Z} y_{ss}^{\alpha_K+\alpha_Z}$$
$$y_{ss} = A \left(\frac{s_Z}{\delta_Z}\right)^{\frac{\alpha_Z}{1-\alpha_Z-\alpha_K}} \left(\frac{s_K}{\delta_K}\right)^{\frac{\alpha_K}{1-\alpha_K-\alpha_Z}}.$$

Steady state

- So far we have setup our model to include unmeasured capital and obtained an equation for steady state output per capita.
- To see if we can account for the differences in per capita GDP between the U.S. and the world's poorest countries, we will choose reasonable parameters for α_K, α_Z, and δ_Z and compare the values of steady state of output per capita we get.
- Recall in our lecture using only physical capital, we were able to account for around $1/3$ of the difference.

Calibration

- The net return for an investment in unmeasured capital is given by the marginal product of unmeasured capital minus depreciation.
- Recall the marginal product of unmeasured capital is given by

$$\begin{aligned} \text{MPZ} &= \alpha_Z A^{1-\alpha_Z-\alpha_K} k_{ss}^{\alpha_K} z_{ss}^{\alpha_Z-1} \\ &= \alpha_Z \frac{y_{ss}}{z_{ss}}. \end{aligned}$$

- Recall in equilibrium we must have

$$\begin{aligned} \delta_Z z_{ss} &= s_Z y_{ss} \\ \frac{\delta_Z}{s_Z} &= \frac{y_{ss}}{z_{ss}}. \end{aligned}$$

- So

$$\text{MPZ} = \alpha_Z \frac{\delta_Z}{s_Z}.$$

Calibration

- We said the net return for an investment in unmeasured capital is the marginal product of unmeasured capital minus depreciation.
- We established
$$\mathsf{MPZ} = \alpha_Z \frac{\delta_Z}{s_Z}.$$
- So the net return is given by
$$\begin{aligned}\mathsf{MPZ} - \delta_Z &= \alpha_Z \frac{\delta_Z}{s_Z} - \delta_Z \\ &= \delta_Z \left(\frac{\alpha_Z}{s_Z} - 1 \right).\end{aligned}$$

Calibration

- Prescott chose the unmeasured capital share $\alpha_Z = 0.5$, physical capital share $\alpha_K = 0.2$, and depreciation of unmeasured capital $\delta_Z = 2.5\%$.
- Supposing a 5% return on net investment for the U.S., we can calculate the investment rate of unmeasured capital to be

$$
\begin{aligned}
.05 &= \delta_Z \left(\frac{\alpha_Z}{s_Z} - 1 \right) \\
&= .025 \left(\frac{.50}{s_Z} - 1 \right) \\
3 &= \frac{.50}{s_Z} \\
s_Z &\approx 16.67\%.
\end{aligned}
$$

Comparison

- Suppose the poorest countries have $10\times$ lower investment in unmeasured capital.
- If keep the unmeasured capital share α_Z, the physical capital share α_K, and depreciation δ_K the same, then this would mean a return on unmeasured capital of

$$\delta_Z \left(\frac{\alpha_Z}{s_Z} - 1 \right) = .025 \left(\frac{.50}{.0167} - 1 \right)$$
$$\approx 72.3\%.$$

- This return is implausibly high.

Comparison

- Suppose instead we see the unmeasured capital investment rate that yields a 25% return in the poorest countries.
- Then we plug in

$$
\begin{aligned}
.25 &= \delta_Z \left(\frac{\alpha_Z}{s_Z} - 1 \right) \\
&= .025 \left(\frac{.50}{s_Z} - 1 \right) \\
11 &= \frac{.50}{s_Z} \\
s_Z &\approx 4.55\%.
\end{aligned}
$$

- This is the unmeasured capital investment rate we'll use when comparing.

Comparison

- We had rough guesses of $s_Z = 16.16\%$ for the U.S. and $s_K = 4.55\%$ for the poorest countries.
- Our formula for steady state output was given by

$$y_{ss} = A\left(\frac{s_Z}{\delta_Z}\right)^{\frac{\alpha_Z}{1-\alpha_Z-\alpha_K}}\left(\frac{s_K}{\delta_K}\right)^{\frac{\alpha_K}{1-\alpha_K-\alpha_Z}}.$$

- Only allowing the unmeasured capital investment rate to differ, we can divide between the U.S. and world's poorest countries, we get that

$$\frac{y_{ss}^{\text{poorest}}}{y_{ss}^{US}} = \left(\frac{4.55}{16.67}\right)^{\frac{0.5}{0.3}} \approx \frac{1}{10}.$$

Comparison

- Per capita U.S. GDP tends to be around 75 times greater than the world's poorest countries.
- To fully explain this using our unmeasured capital, we'd need the unmeasured capital investment rate to be so low the returns to unmeasured capital would be completely unreasonable.
- After expanding our definition of investment to be any payment now that gives returns in the future, we still see that TFP is important in cross-country differences.

Sources of TFP Differences

- Corruption and governance issues: Rampant corruption and stifling bureaucracies can lead to misallocated resources and discourage work.
- Political instability or conflict: War, civil unrest, and unpredictable regime changes are all events that cause fear and uncertainty, leading to reduced investment.
- Weak property rights: Fear of expropriation or an inability to enforce contracts can lead to reduced lending.
- Infrastructure and public goods: Poor public infrastructure and health can drive poor productivity.

Next time

- So far we've thought of L to be the number of workers in the country.
- Human capital allowed us to think of workers of different countries having different productivities, but we have not yet considered that workers in different countries might work at different hours.
  - An average worker in the U.S. needs to work less hours to consume the same amount than they needed to in 1960, does this mean workers work less and take more time off, or more since work is comparatively more rewarded?
- When discussing the Production model, we took labor supplied and capital supplied to be exogenous.
- In the next lecture we will endogenize labor supplied.