Labor Supply

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Review

- Consider the profit maximization problem

$$\max_{K,L} AK^{\alpha}L^{1-\alpha} - RK - WL.$$

- In our production model, we knew that the labor and capital demanded must satisfy

$$\begin{aligned} \text{MPL} &= W \\ (1-\alpha)AK^{\alpha}L^{-\alpha} &= W \end{aligned}$$

$$\begin{aligned} \text{MPK} &= R \\ \alpha AK^{\alpha-1}L^{1-\alpha} &= R. \end{aligned}$$

- The firm demanded capital and labor until the marginal product of capital equaled the interest rate (R) and the marginal product of labor equaled the real wage (W).

Review

- We had equations that governed the demand for labor.
- For the supply of labor, we simply took it as given that there was capital $\bar{K}$ and labor $\bar{L}$ that was supplied.
- Recall we had a market clearing condition, the idea that the labor demanded equals labor supplied, which meant $K = \bar{K}$ and $L = \bar{L}$.
- In this lecture we will model how labor is supplied in an economy.
- You will model how capital is supplied in a future lecture.

Hours Worked

- Until now, labor supplied $\bar{L}$ has been taken as given.
- We have not considered the wage workers are being paid.
  1. What if a higher wage makes you **more** likely to work, since working is rewarded more to other alternatives?
  2. What if a higher wage makes you **less** likely to work, since it takes less work to earn the same income?
- To model labor supply, we'll need a model that incorporates individual's decision-making.

Micro Refresher

- Recall an individual's utility maximization problem from your intermediate micro course

$$\max_{x_1,x_2} u(x_1, x_2)$$

$$\text{such that } p_1x_1 + p_2x_2 = y.$$

- Given a total budget y, you choose a bundle of goods (x_1, x_2) that both maximize your utility and is affordable with prices (p_1, p_2).

Utility Maximization

- Consider the following problem:

$$\max_{c,n} u(c) - v(n)$$
$$\text{such that } c = wn$$

- Taking the wage w as given, an individual chooses consumption (c) and labor supplied (n) that maximizes their utility while satisfying their budget constraint.

Utility Maximization

- We had

$$\max_{c,n} u(c) - v(n)$$

$$\text{such that } c = wn.$$

- An assumption here is that the real wage w is taken as given. If the individual chooses to work more hours, it will not raise the real wage (w).
- We're also assuming an individual chooses to spend all of their income. This is a one-period model, so there's no idea of saving for tomorrow.

Utility Maximization: Consumption

- Economists model consumption as the goods and services a person enjoys. Examples include food, transportation, entertainment, and clothing.
- Economists assume $u(c)$ is strictly increasing. That is, the more you consume, the better off you are.
- To consume more, people need to work more, which is why $u(c)$ is paired with $v(n)$, giving us a tradeoff between the disutility from working and the utility from consuming.

Utility Maximization: Labor

- Recall the utility an agent received from working n hours was $-v(n)$.
- We assume working gives agents disutility. That is, the household in our model would prefer a world where they don't work at all and consume an infinite amount of goods.
- We assume $v(n)$ is strictly increasing in n. The more an agent works, the larger the disutility from working.
- The budget constraint $c = wn$ forces the household to work in order to consume.

Solution: Existence

- We had

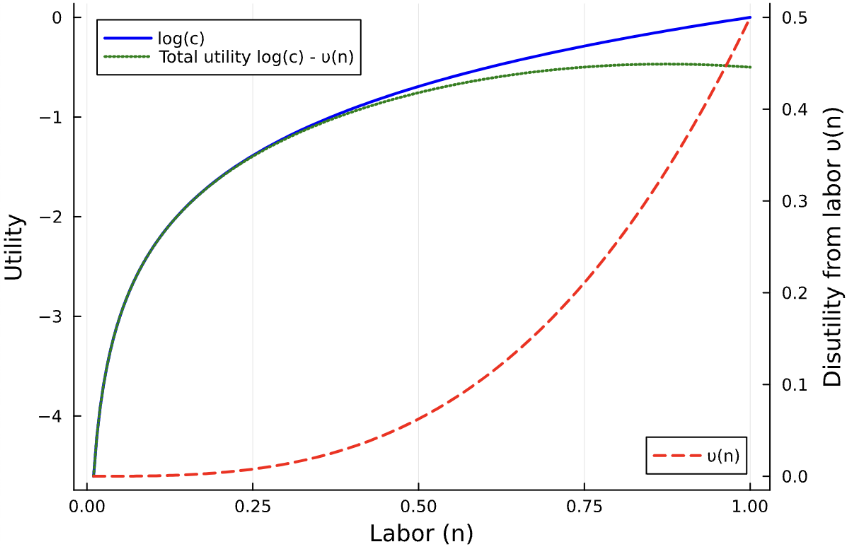
$$\max_{c,n} u(c) - v(n)$$
$$\text{such that } c = wn.$$

- A solution to this problem will be a value for consumption and labor (c, n) that maximizes the above problem while satisfying the budget constraint.
- How do we know there is a solution? Since there's no upper bound on hours worked, what's to prevent the household from working infinite hours and consuming an infinite amount?

Solution: Existence

- We had the budget constraint
$$c = wn.$$
- If we know labor (n) then we automatically know consumption (c) since we take the real wage (w) as given.
- A typical utility specification people use is
$$\max_{c,n} \log(c) - \chi \frac{n^{1+\frac{1}{\varepsilon}}}{1+\frac{1}{\varepsilon}}$$
such that $c = wn$.
- We will plot utility values $u(c)$ and labor disutility $v(n)$ for different levels of labor n where $c = wn$.

Solution: Existence[1]



[1] Solution with utility specification from previous slide with $w = 1.0$, $\chi = 1.5$, and $\varepsilon = 0.5$

Solution: Existence

- Notice that the increase in utility that came from an increase in consumption was inversely related to the level of consumption.
- That is, the more you consumed, the smaller the benefit from an additional unit of consumption.
- On the labor side, we saw the opposite.
- The more you worked, the bigger the penalty from an additional unit of work.
- This guarantees us a solution, as eventually the cost of working is so high it outweighs the benefit from consuming from working more.

MRS

- The marginal rate of substitution tells us how much more consumption an agent needs to be just as well off after working a little more.
- Let $U(c, n) = u(c) - v(n)$, then the marginal rate of substitution is given by

$$\text{MRS} = -\frac{U_n}{U_c}.$$

- At the optimal point, the MRS must equal the real wage (w)

$$-\frac{U_n}{U_c} = w.$$

- At the optimal point, the marginal disutility from work must equal the marginal utility from consumption times the extra consumption an additional increase in work can buy.
- See the appendix for a derivation of this equality. See Appendix

Example: Solution

- Now consider the example:

$$\max_{c,n} \log(c) - \frac{\chi}{1+\frac{1}{\varepsilon}} n^{1+\frac{1}{\varepsilon}}$$

such that $c = wn$.

- Recall our equation for the MRS,

$$-\frac{U_n}{U_c} = w.$$

- Filling in these equations with their values from the problem above, we get

$$\frac{\chi n^{\frac{1}{\varepsilon}}}{\frac{1}{c}} = w.$$

Example: Solution

- We used the MRS to get

$$\frac{\chi n^{\frac{1}{\varepsilon}}}{\frac{1}{c}} = w$$

  and have our budget constraint

$$c = wn.$$

- We have two equations with two unknowns.
- First, use the MRS to get consumption in terms of labor n,

$$\frac{\chi n^{\frac{1}{\varepsilon}}}{\frac{1}{c}} = w$$

$$c = \frac{w}{\chi n^{\frac{1}{\varepsilon}}}.$$

Example: Solution

- Next, plug in our value for consumption into the budget constraint

$$
\begin{aligned}
c &= wn \\
\frac{w}{\chi n^{\frac{1}{\varepsilon}}} &= wn \\
\frac{1}{\chi} &= n^{1+\frac{1}{\varepsilon}} \\
n &= \left(\frac{1}{\chi}\right)^{\frac{\varepsilon}{\varepsilon+1}}.
\end{aligned}
$$

- That is, with the utility function above and taking wage w as given, the household supplied $n = \left(\frac{1}{\chi}\right)^{\frac{\varepsilon}{\varepsilon+1}}$ labor.

Labor Supplied

- We had

$$n = \left(\frac{1}{\chi}\right)^{\frac{\varepsilon}{\varepsilon+1}}.$$

- Recall one of the questions we asked is if higher wages made the household work more or work less.
- According to the above formula, the household works the same no matter the wage.
- This is because of the utility function we chose for consumption (log utility), we will see other utility functions that do not have this property.

Labor Supplied

- We wanted to ask the question if an increase in the real wage would lead to an increase or decrease in labor supplied.
- As the real wage increases, labor is now rewarded more, so individuals substitute away from leisure and more towards labor. This is called the **substitution effect**.
- As the real wage increases, an individual now has to work a smaller number of hours to earn the same income they did before and maintain the same standard of living. An individual works less and spends more of their time in leisure, this is called the **income effect**.

Example II

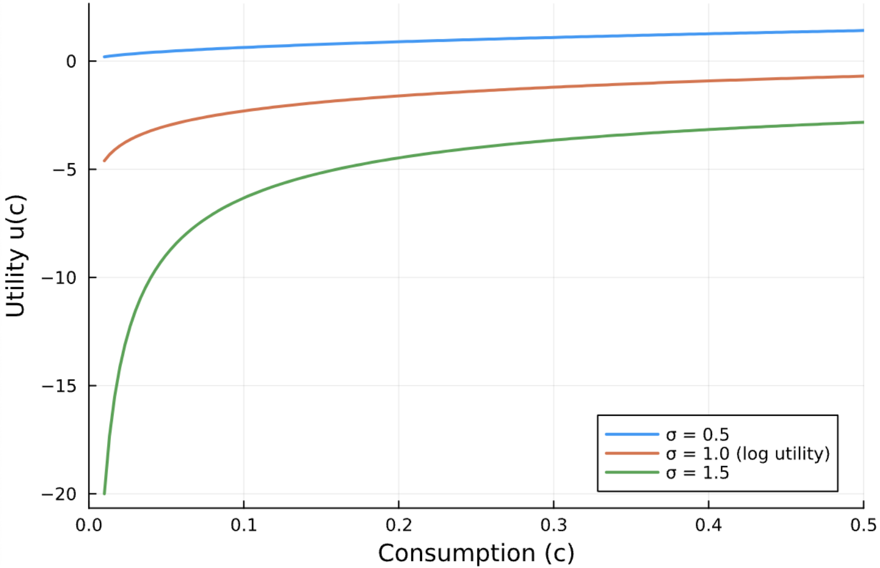
- We now will consider an example where a change in the real wage (w) **does affect** a change in labor supplied (n).
- Consider the same utility maximization problem with a different utility function

$$\max_{c,n} \frac{c^{1-\sigma}}{1-\sigma} - \frac{\chi}{1+\frac{1}{\varepsilon}} n^{1+\frac{1}{\varepsilon}}$$

such that $c = wn$.

- This utility function is a generalization of log utility. It can be shown when $\sigma = 1$ this utility function becomes log utility.

Example II: σ Utility



Example II: MRS

- We said earlier that the optimal pair (c, n) will satisfy

$$\text{MRS} = -\frac{U_n}{U_c} = w.$$

- Using this on our utility function

$$\frac{c^{1-\sigma}}{1-\sigma} - \frac{\chi}{1+\frac{1}{\varepsilon}} n^{1+\frac{1}{\varepsilon}}$$

gets us

$$\frac{\chi n^{\frac{1}{\varepsilon}}}{c^{-\sigma}} = w.$$

Example II: Budget Constraint

- Our MRS equation told us

$$\frac{\chi n^{\frac{1}{\varepsilon}}}{c^{-\sigma}} = w.$$

- We can rearrange to get[2]

$$c = \left(\frac{w}{\chi n^{\frac{1}{\varepsilon}}}\right)^{\frac{1}{\sigma}}.$$

- Like before, we know the optimal pair (c, n) will satisfy this equation.

[2] Earlier we said when $\sigma = 1$ this is log utility. Note that when $\sigma = 1$ our equation for c is exactly like what we had before.

Example II: Solution

- We had
$$c = \left(\frac{w}{\chi n^{\frac{1}{\varepsilon}}}\right)^{\frac{1}{\sigma}}.$$
- We need to find the pair (c, n) that not only solves our optimality condition but is affordable.
- We can use the budget constraint to get
$$\begin{aligned} c &= wn \\ \left(\frac{w}{\chi n^{\frac{1}{\varepsilon}}}\right)^{\frac{1}{\sigma}} &= wn \\ n &= \left(\frac{w^{1-\sigma}}{\chi}\right)^{\frac{\varepsilon}{1+\sigma\varepsilon}} \end{aligned}$$

Elasticity

- We have
$$n = \left(\frac{w^{1-\sigma}}{\chi}\right)^{\frac{\varepsilon}{1+\sigma\varepsilon}}.$$
- Question: How does a change in the wage (w) affect labor supplied (n)?
- Answer: Let's calculate the elasticity of labor supplied (n) with respect to the real-wage (w).
- Recall that using our formula for an elasticity here means
$$\varepsilon_{n,w} = \frac{d\log n}{d\log w}.$$

Elasticity

- We have
$$n = \left(\frac{w^{1-\sigma}}{\chi}\right)^{\frac{\varepsilon}{1+\sigma\varepsilon}}.$$
- This means
$$\ln(n) = \frac{\varepsilon(1-\sigma)}{1+\sigma\varepsilon}\ln(w) - \frac{\varepsilon}{1+\sigma\varepsilon}\ln(\chi).$$
- Using our formula for the elasticity of the real wage (w) with respect to labor supplied (n) gives us
$$\varepsilon_{n,w} = \frac{d\ln(n)}{d\ln(w)} = \frac{\varepsilon(1-\sigma)}{1+\sigma\varepsilon}.$$

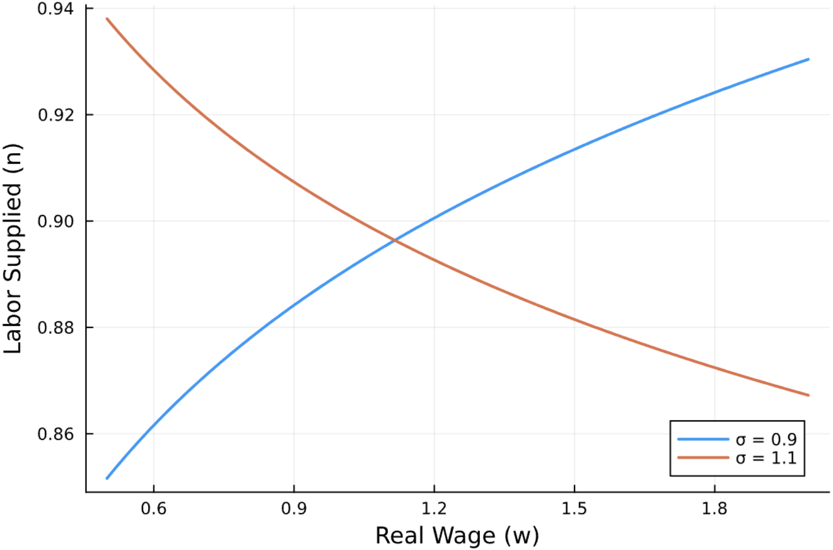
Elasticity

- We calculated the elasticity of labor supplied (n) with respect to the real wage (w) to be

$$\varepsilon_{n,w} = \frac{\varepsilon(1-\sigma)}{1+\sigma\varepsilon}.$$

- This means a 1% increase in the real wage w implies a $\frac{\varepsilon(1-\sigma)}{1+\sigma\varepsilon}\%$ increase/decrease in labor supplied (n).
- Whether or not the effect increases or decreases labor supplied comes from whether or not $\sigma < 1$.
  - If $0 < \sigma < 1$, labor supplied (n) **increases** with an increase in the real wage (w).
  - If $\sigma > 1$, then labor supplied (n) **decreases** with an increase in the real wage (w).

Income and Substitution Effects[3]



[3]Parameters $\chi = 1.1, \varepsilon = 1.5$ are held constant.

Income and Substitution Effects

- In our first example with log utility, we found

$$n = \left(\frac{1}{\chi}\right)^{\frac{\varepsilon}{\varepsilon+1}}.$$

- That is, hours worked (n) was not impacted by the real wage (w).
- This means the magnitudes of the income and substitution effects were equal, so the effects canceled each other out.

Income and Substitution Effects

- In our second example, we found

$$n = \left(\frac{w^{1-\sigma}}{\chi} \right)^{\frac{\varepsilon}{1+\sigma\varepsilon}}.$$

- When $0 < \sigma < 1$, we found labor supplied (n) **increased** with an increase in the real wage (w). This means the substitution effect was larger than the income effect.
- When $\sigma > 1$, we found labor supplied (n) **decreased** with an increase in the real wage (w). This means the income effect was larger than the substitution effect.

Income and Substitution Effects

- We saw in previous lectures that in the past several decades GDP per capita has increased around the world.
- As incomes rose, did substitution effects or income effects dominate?
- As incomes rise, if the income effect dominates then we'd expect to see hours worked fall across countries.
- As incomes rise, if the substitution effect dominates, then we'd expect to see hours worked increase across countries.

Income and Substitution Effects[4]

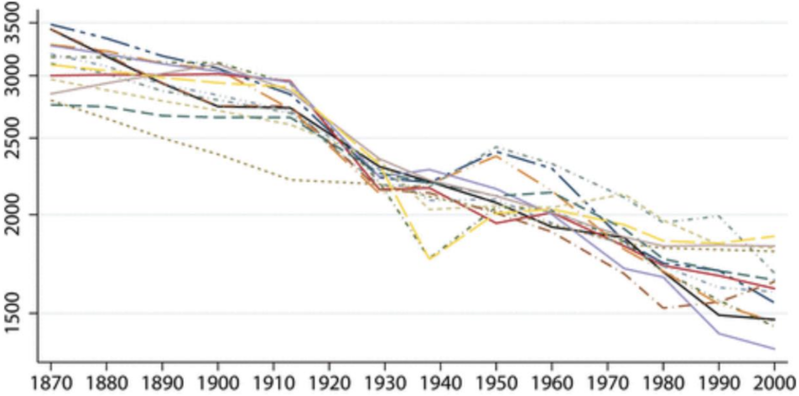


FIG. 1.—Hours worked per worker. The figure shows data for the following countries: Belgium, Denmark, France, Germany, Ireland, Italy, the Netherlands, Spain, Sweden, Switzerland, the United Kingdom, Australia, Canada, and the United States. The scale is logarithmic, which suggests that hours fall at roughly 0.57% per year. Source: Huberman and Minns (2007). Maddison (2001) shows a similar systematic decline in hours per capita.

[4] Source: Timo Boppart and Per Krusell, "Labor Supply in the Past, Present, and Future: A Balanced-Growth Perspective", JPE, 2020

Moving Forward

- Now that we have a model of labor supplied, can we use it to explain labor supply differences across countries?
- Do taxes in income affect the labor supply?
- If so, how would we incorporate them into our model?
- Many individuals receive transfers from the government (an example being social security), how would we incorporate these into our model?

Appendix: Deriving MRS

- In economics, we often use Lagrangians to solve maximization problems with budget constraints.
- This helps us turn a maximization problem into an algebra problem in a mechanical and widely applicable way.
- Suppose we're solving

$$\max_{c,n} \; u(c) - v(n)$$
$$c = wn.$$

- The Lagrangian for this problem is given by

$$\mathcal{L} = u(c) - v(n) - \lambda(c - wn).$$

  That is, the function we're trying to maximize minus λ (an unknown number) times the constraint.

Appendix: Deriving MRS

- Once you have the Lagrangian, the next step is to take the derivative with respect to each endogenous variable and setting them equal to zero. Doing this for consumption and labor yields

$$
\begin{aligned}
[c]: &\quad u'(c) - \lambda = 0 \\
[n]: &\quad v'(n) - \lambda w = 0.
\end{aligned}
$$

- Note that we have this unknown number λ, however, if we divide both sides we can calculate

$$
\begin{aligned}
\frac{v'(n)}{u'(c)} &= \frac{\lambda w}{\lambda} \\
\frac{v'(n)}{u'(c)} &= w
\end{aligned}
$$

which is exactly the MRS equation we said above. Back